Available online at www.sciencedirect.com

ScienceDirect

Chemical Engineering and Processing 47 (2008) 251–264

Chemical Engineering and Processing
Process Intensification

www.elsevier.com/locate/cep

Optimal tuning of PID controllers for FOPTD, SOPTD and SOPTD with lead processes

C.R. Madhuranthakam, A. Elkamel *, H. Budman

Department of Chemical Engineering, University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada

Received 21 October 2006; received in revised form 27 November 2006; accepted 28 November 2006
Available online 3 December 2006

Abstract

This paper presents the synthesis and analysis of optimal tuning of proportional integral derivative (PID) parameters for different process systems: first order plus time delay (FOPTD), second order plus time delay (SOPTD) and second order plus time delay with lead (SOPTDLD). This work involved optimization of the PID control parameters to achieve the minimization of the integral absolute error (IAE). A set of new and generalized tuning correlations relating the controller parameters to the process parameters was obtained for step changes in set point and load separately. Simulation results showed that the use of the proposed tuning correlations results in superior closed loop performance compared to other tuning techniques previously proposed in the literature.

Keywords: PID control; FOPTD; SOPTD; SOPTD with lead; Multi-objective optimization; IAE; Ziegler–Nichols method; Cohen–Coon method

1. Introduction

Proportional integral derivative (PID) controllers are still widely used today to control many processes because of their long history of proven operation in addition to the fact that they are well understood by many operational, technical and maintenance personnel. Furthermore, an industrial PID controller has many extensions that make it a practical tool for operating a chemical process. Many PID controller tuning methods have been proposed in the literature. For example, Ziegler–Nichols tuning [1], Cohen–Coon tuning [2], Direct Synthesis Method [3], Internal Model Control [4], tuning rules based on the minimization of different error criteria [5], and neural networks based methodologies [6,7]. All these methods have their own advantages, disadvantages and limitations. Most of the tuning methods were proposed for first order plus time delay (FOPTD) system, as they can explain the behavior of a wide range of processes. Thus, suitable tuning rules for PID controllers are needed for processes represented by transfer functions other than FOPDT. Also, model based tuning methods were found attractive for practitioners because they have only one tuning parameter. However, many control systems do not provide Smith Predictor or Internal Model Control functionality [8]. At the same time, many distributed parameter systems where a state variable is a function of more than one independent variable encountered in the process plants such as packed beds, trickle beds with recycle, continuously stirred tank reactors (CSTRs) in series, etc., cannot be modeled as FOPTD and instead, are often empirically modeled using second order plus time delay (SOPTD) and SOPTD with lead (SOPTDLD). Many industrially proven techniques are now available for fitting these models to plant data [9].

In this paper, a set of tuning correlations is obtained for three different processes whose dynamics are modeled with FOPTD, SOPTD and SOPTDLD. Most of the chemical processes can be effectively modeled by one of these types of models. In this work, controller tuning parameters that minimize integral absolute error (IAE) are investigated. The resulting solutions are then summarized in the form of algebraic correlations that can be conveniently used by practitioners when applying PID control. In the proposed method, the controller parameters obtained from

Abbreviations: C–C, Cohen and Coon; CSTR, continuously stirred tank reactor; FOPTD, first order plus time delay; IAE, integral of absolute error; ISE, integral of square error; ITAE, integral of time weighted absolute error; ITSE, integral of time weighted square error; PID, proportional integral derivative; P–M, proposed method; R/L, Rovira (1981)/Lopez et al. (1967); SOPTD, second order plus time delay; SOPTDLD, second order plus time delay with lead; Z–N, Ziegler–Nichols

* Corresponding author. Tel.: +1 519 888 4567; fax: +1 519 746 4979.
E-mail address: aelkamel@uwaterloo.ca (A. Elkamel).

0255-2701/$ – see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.cep.2006.11.013

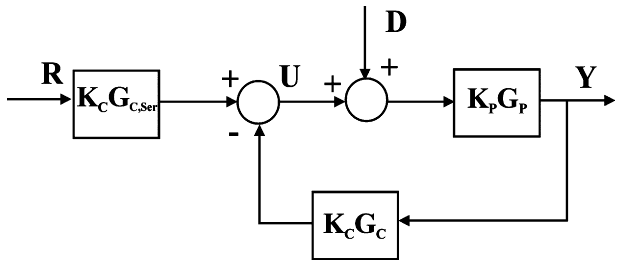

Fig. 1. Schematic of the feedback control loop.

Ziegler–Nichols' method are used as the initial guess for the optimization problem that deals with the minimization of the IAE in the output variable.

2. Proposed tuning correlations

The ideal PID controller transfer function is defined by Eq. (1):

$$K_C G_C = K_C \left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \tag{1}$$

The above transfer function is not physically realizable since no device can be constructed which truly differentiates an input signal (Luyben and Luyben [10]). Also, an ideal PID controller causes a derivative kick when used in the feedback control system especially for a step change in the set point. Thus, an implemental form of the PID controller that is based on augmenting Eq. (1) with a first order filter of the derivative term is used instead. This can be achieved either by a series or parallel configuration. Most often, the derivative term multiplied by the filter transfer function is used on the measured variable rather than on the controller error. This is done to avoid a derivative kick occurring following step changes in the set point. For the simulations conducted in the present manuscript, a parallel form of PID controller configuration as shown in Fig. 1 was used. The transfer function for the parallel form PID controller is given by Eq. (2).

$$U(s) = K_C \left(\frac{\tau_I s + 1}{\tau_I s}\right) e(s) - K_C \left(\frac{\tau_D s}{\alpha \tau_D s + 1}\right) Y(s) \tag{2}$$

where U is the controller output, e the error and Y is the controlled variable as shown in Fig. 1. The "derivative filter factor" α value is generally selected in the interval [0.05, 0.2] (Shinskey [11] and Luyben [12]) and most often it is preset to 0.1. In the current work, a value of $\alpha = 0.1$ is used for conducting the comparison of the performance of the proposed tuning method for different process transfer functions. Fig. 1 shows the conventional feedback control system used in the present study. In Fig. 1, $K_C G_{C,Ser}$ is the servo compensator ($=K_C(1 + (1/\tau_I s))$), $K_P G_P$ the process, $K_C G_C$ the controller ($=K_C(1 + (1/\tau_I s) + (\tau_D s/(\alpha \tau_D s + 1)))$), R the set point, U the controller output, D the disturbance variable and Y is the controlled variable.

The combined dynamics of the process, final control element and the sensor is assumed to be conveniently represented by FOPTD, SOPTD and SOPTDLD. The corresponding transfer functions for the processes are described by the following equations:

FOPTD:

$$K_P G_P = \frac{K_P \exp(-\theta s)}{\tau_1 s + 1} \tag{3}$$

SOPTD:

$$K_P G_P = \frac{K_P \exp(-\theta s)}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{4}$$

SOPTDLD:

$$K_P G_P = \frac{K_P(\tau_3 s + 1) \exp(-\theta s)}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{5}$$

The error $e(t)$ is defined by Eq. (6) given below:

$$e(t) = R(t) - Y(t) \tag{6}$$

Ciancone and Marlin [13] have recently made an attempt at developing simple tuning rules for FOPTD processes with the following goals in mind: (1) minimization of the IAE, (2) assumption of a +25% (correlated) change in the process model parameters and (3) compliance with pre-specified limits on the variation of the manipulated variable. They proposed different charts for dimensionless tuning constants in terms of the fraction dead time defined as $\theta/(\theta + \tau_1)$, with the above constraints. Separate charts were obtained by these authors for both set point and load changes. In their method, optimization was done by continuously varying the controller parameters until the objective function was satisfied by the above constraints. Since the objective function for minimizing all the constraints was non-linear, they proposed to optimize each controller parameter independently as per the following procedure:

1. With the integral time set to infinity and derivative time set to zero, they obtained an optimal proportional gain.
2. Using the optimal proportional gain obtained in step 1, they optimized the integral time followed by the optimum derivative time.

In the present work, we attempt to develop simple and useful mathematical expressions that can be readily used for tuning an extensive set of processes and as described by the transfer functions (3)–(5). In order to improve the convergence properties of the involved optimization problems in search of the tuning parameters, the following steps are employed:

1. For a unit step change in set point or load, with proportional control alone, the ultimate gain and ultimate period (the gain for which the system is at the limit of stability and the corresponding period of oscillation) for a particular process transfer function (FOPTD, SOPTD and SOPTDLD) is determined.
2. Ziegler–Nichols tuning rules are used to provide an initial guess for the non-linear optimization. The best controller

tuning parameters are then obtained with the objective of minimizing IAE.
3. Steps 1 and 2 are repeated for various different combinations for the process parameters, τ_1, τ_2, τ_3, K_P and θ. All process parameters are varied from 1 to 50.
4. The controller parameters thus obtained are made dimensionless by multiplying/dividing by appropriate scale factors.
5. Using regression techniques, simple and accurate correlations are obtained for the controller parameters as functions of process parameters (in terms of τ_1, τ_2, τ_3 and θ) for the corresponding three processes. Several sets of dimensionless groups are tried for the tuning rules and the ones that lead to the largest coefficients of correlations (R^2) are retained in the proposed tuning rules.

The IAE is defined as:

$$\mathrm{IAE} = \int_0^\infty |e(t)|\,\mathrm{d}t \tag{7}$$

where $e(t)$ is error defined according to Eq. (6). Though the upper bound on time is infinity, in the simulations the integration was performed over a sufficiently long time as compared to the closed loop settling time, i.e. after the response reaches a steady state. Minimum IAE was chosen as a suitable objective function in the optimization because it is an indication of product variability. Minimization of the IAE generally produces intermediate responses between fast responses obtained from the minimization of the integral square error (ISE) and slow responses obtained from the minimization of the integral time weighted absolute error (ITAE) [14]. The process models were designed and simulated using SIMULINK and the dynamic optimization was performed using a combination of the SIMULINK results with MATLAB programs used to calculate the IAE and to interface the results with the optimization routines. Separate tuning rules for set point change and load change were developed for the different processes. The optimized controller parameters thus obtained were correlated with the process parameters according to the following equations:

$$K_C = f_1(K_P, \tau_1, \tau_2, \tau_3, \theta) \tag{8}$$

$$\tau_I = f_2(K_P, \tau_1, \tau_2, \tau_3, \theta) \tag{9}$$

$$\tau_D = f_3(K_P, \tau_1, \tau_2, \tau_3, \theta) \tag{10}$$

In Eqs. (8)–(10), the unit of θ, τ_1, τ_2, τ_3 is time and the unit of K_P depends on the plant input and output. In principle, the number of possible independent dimensionless numbers that can be formed for the purpose of representing the tuning rules is very high. Analysis of control performance using the dimensionless groups obtained by using the Pi-theorem for the FOPTD and SOPTD (for load change in terms of charts) models can be found elsewhere (Sayeed and Mahdi [15] and Archibald and Tae-Won [16]).

In the present work, to avoid the multiple charts and formulae for the controller parameters of different processes considered, inspection analysis was performed to get the dimensionless controller parameters. This inspection procedure is made of the following two steps:

Table 1
First moments (time-scale) for different processes

Process ($K_P G_P$)	Time-scale	Fraction dead time
FOPTD	$\theta+\tau_1$	$\theta/(\theta+\tau_1)$
SOPTD	$\theta+\tau_1+\tau_2$	$\theta/(\theta+\tau_1+\tau_2)$
SOPTDLD	$\theta+\tau_1+\tau_2-\tau_3$	$\theta/(\theta+\tau_1+\tau_2-\tau_3)$

1. The independent variable for the tuning correlations was selected as the fraction dead time, i.e. the ratio of the dead time of the process and the sum of all the time constants including the dead time.
2. The dependent variables for each process for set point or load change (separately) was obtained by continuously iterating the different possible dimensionless groups as a function of τ_I and τ_D with different combinations of the process parameters (θ, τ_1, τ_2, τ_3) until a high degree of correlation, i.e. with $R^2 \geq 0.95$ between the independent and the dependent variables were obtained. The tuning correlations obtained from the simulations have an R^2 value in the range of 0.97–0.99.

To simplify the correlations further, a time scaling was used in our proposed method. Every process responds with a different "speed", which can be characterized by the time for a step response to achieve 63% of its final value. This is also known as the first moment of the impulse response. For example, the method of moments was previously used to find the characteristic time of the process (Marlin [22]). In this work, the first moment was used to normalize the dynamic responses with respect to time in the dimensional analysis. Hence, for the processes considered in the study, the first moment or the time-scale is shown in Table 1. Dividing all time constants by this time-scale will normalize all the models by the same speed of response. The range of the variables K_P, θ, τ_1, τ_2 and τ_3, used in the simulations was 0–50 (for K_P it was 1–50, for the SOPTLD all parameters are varied from 1 to 20). It was found that there was no clear relation between the fraction dead time and the scaled integral and derivative terms. Hence, the integral and derivative parameters are scaled with respect to the process time constants by trial and error to get a simple algebraic correlation with a high R^2. The performances of different processes with the proposed tuning methods were compared with the conventional tuning methods.

3. Simulation results and analysis

3.1. FOPTD

Simulations were performed by varying the dead time and the time constant in the range from 1 to 50. The ratio of the dead time to the time constant of the process (θ/τ_1) corresponding to the above settings varied from 0.1 to 2. The tuning parameters thus obtained for a step change in set point and load were separately correlated with the fraction dead time. Fig. 2 shows the comparison of the results obtained from the simulations and

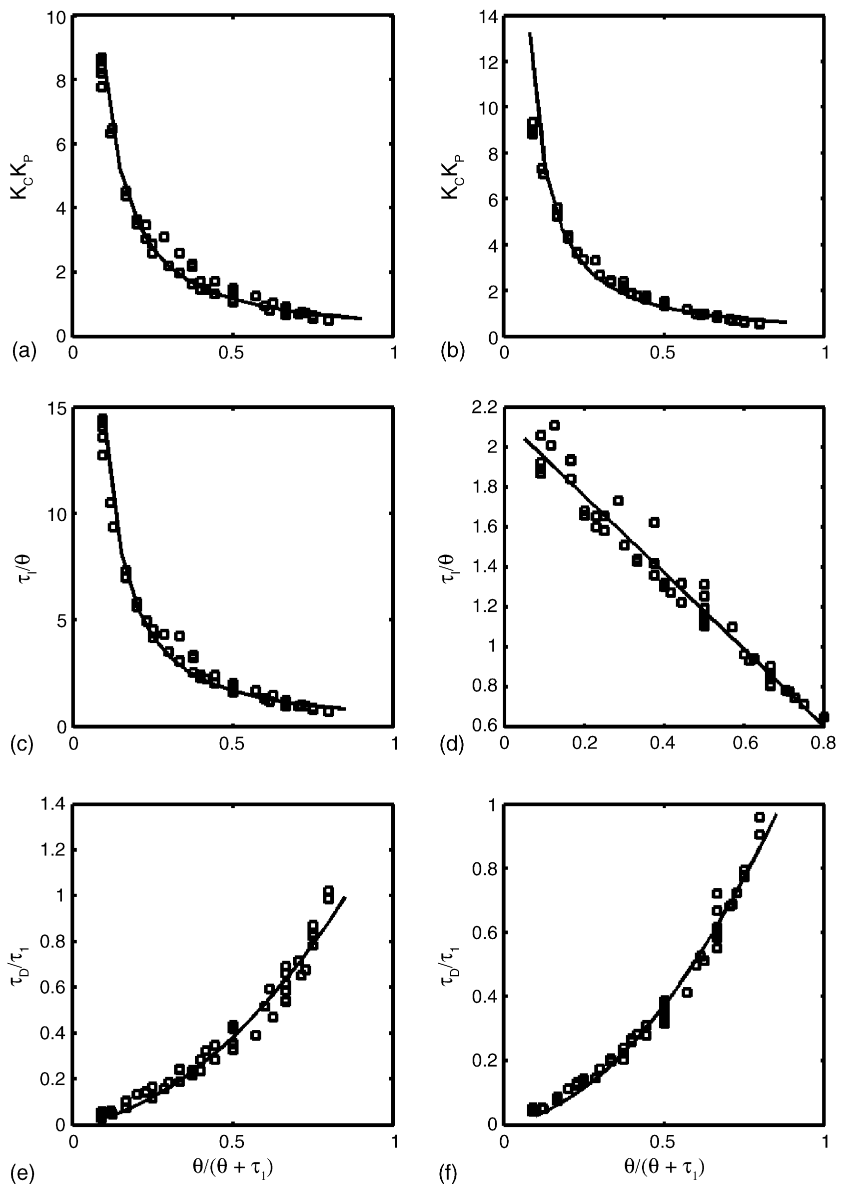

Fig. 2. Dimensionless control parameters obtained from model vs. simulation results for first order plus time delay process (described by Eq. (3)): (a, c and e) for set point change and (b, d and f) for load change.

the tuning model. Fig. 2(a, c and e) correspond, respectively, to the proportional gain, integral time and derivative time relationship for a unit step change in the set point. Fig. 2(b, d and f) are the correlations of the tuning parameters corresponding to a unit step change in the load. The tuning models are shown in Table 2. Rovira [17] developed tuning rules with the same objective function, i.e. minimization of IAE. His tuning rules can be used with θ/τ_1 varying from 0.1 to 1. For processes with a ratio θ/τ_1 greater than one, the performance was not robust and gives an over stable performance. This is further discussed in the following case studies section. Also, Lopez et al. [18] developed separate tuning rules for FOPTD processes for unit

Table 2
Proposed tuning relations for FOPTD process

Tuning parameter	Set point change	Load change
K_C	$\dfrac{0.4967}{K_P}\left(\dfrac{\theta}{\theta+\tau_1}\right)^{-1.2299}$	$\dfrac{0.5249}{K_P}\left(\dfrac{\theta}{\theta+\tau_1}\right)^{-1.2787}$
τ_I	$0.6739\theta\left(\dfrac{\theta}{\theta+\tau_1}\right)^{-1.317}$	$\theta\left(-1.9167\left(\dfrac{\theta}{\theta+\tau_1}\right)+2.1356\right)$
τ_D	$\tau_1\left(1.138\left(\dfrac{\theta}{\theta+\tau_1}\right)^2+0.1992\left(\dfrac{\theta}{\theta+\tau_1}\right)\right)$	$\tau_1\left(1.1321\left(\dfrac{\theta}{\theta+\tau_1}\right)^2+0.1788\left(\dfrac{\theta}{\theta+\tau_1}\right)\right)$

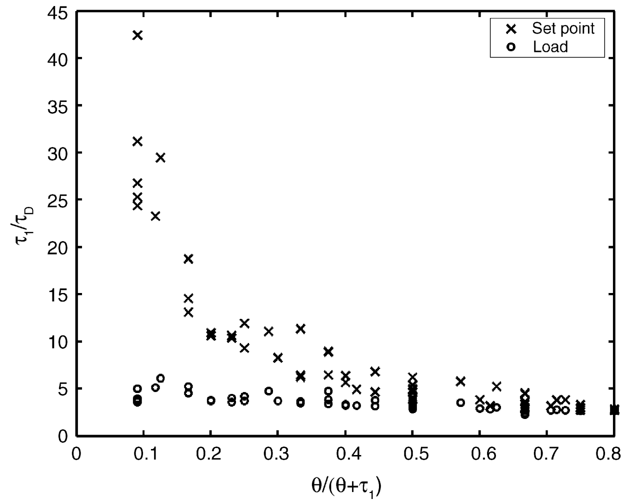

Fig. 3. The ratio between integral time and derivative time as a function of fraction dead time for FOPTD process.

step change in the load variable. In their work, a similar restriction was imposed on the range of θ/τ_1 ratio. Recently, Sayeed and Mahdi [15] developed tuning rules for the FOPTD process with a derivative filter (used on the controller error) in the PID controller. But, the controller parameters obtained from their rules give unstable response when the configuration given by Eq. (2) was used for the PID controller. In the present work, tuning models were developed for θ/τ_1 varying from 0.1 to 2 for the parallel form of the PID controller. The general conclusions for the models obtained are similar to those of Ziegler–Nichols, Rovira [17] and Lopez et al. [18]. Thus, the controller gain is inversely proportional to the process gain. Also, the optimum controller gain decreases with an increase in the θ/τ_1 ratio. The controller gain needed for a load change is slightly greater than the gain needed for a set point change which is evident from the coefficients in the corresponding models shown in Table 2. Furthermore, with an increase in the dead time the integral time decreases and the derivative time increases.

The ratio of integral time to derivative time is very important in assessing the relative importance of the derivative time with respect to integral time. Many PID controllers using a series configuration, work with a τ_I/τ_D greater than 4 (Åstrom and Hägglund [19]). Fig. 3 shows the τ_I/τ_D versus fraction dead time for both set point and load changes. There is a significant variation in the τ_I/τ_D ratio at lower fraction dead time. The figure clearly shows that for $0.1<\theta/(\theta+\tau_1)<0.8$ the τ_I/τ_D ratio obtained from the proposed tuning rules stays at a constant value of 5 ± 2 for load changes. On the other hand, for set point change and for processes without a significant dead time the ratio increases with a decrease in dead time. A similar trend for set point change was found by Rovira [17] with $\tau_I/\tau_D=32$ for $\theta/\tau_1=0.1$.

3.2. FOPTD case studies

To compare the performance of the proposed tuning method with other conventional methods, three different FOPTD processes were considered. The processes are represented by $G_{P1}(s)$, $G_{P2}(s)$ and $G_{P3}(s)$ and are described by Eqs. (11)–(13), respectively.

$$K_{P1}G_{P1}(s)=\frac{1}{5s+1}\exp(-s) \tag{11}$$

$$K_{P2}G_{P2}(s)=\frac{4}{10s+1}\exp(-10s) \tag{12}$$

$$K_{P3}G_{P3}(s)=\frac{1}{7s+1}\exp(-14s) \tag{13}$$

In the process $G_{P1}(s)$, the ratio of the dead time to time constant is 0.2, for $G_{P2}(s)$ the ratio is unity, while the last process $G_{P3}(s)$ has a ratio equal to 2.

Fig. 4 shows the closed loop response due to a set point change and a load change for $G_{P1}(s)$, $G_{P2}(s)$ and $G_{P3}(s)$ processes using different tuning methods. It clearly shows that the Z–N and C–C tuning parameters would give more oscillations before reaching the steady state value thus resulting in a higher IAE. For processes $G_{P1}(s)$ and $G_{P2}(s)$, the response obtained from Rovira [17] was similar to that of the response obtained by using the proposed tuning rules. For load changes, similar results to the ones obtained with the method proposed by Lopez et al. [18] were calculated, though the IAE's obtained with the proposed method were less than those with Lopez method. However, in the case of large dead time ratio (i.e. with the process $G_{P3}(s)$), the proposed method of tuning parameters gave a better response than the other methods. The Z–N and C–C methods resulted in highly oscillatory responses and the other two methods did not perform well too. The controller parameters obtained from different methods and the corresponding performances are shown in Table 3.

In Table 3, T_r is the rise time, T_s the settling time (based on time required to reach 5% of the steady state value) and Os is the overshoot for different servo responses. The overshoot was very low compared to the other methods when the proposed tuning relations are used for designing the PID parameters. Also, the settling time obtained by using the proposed tuning relations was reduced by two to three times as compared to those obtained with the other tuning methods.

3.3. SOPTD

Many process systems may be described by second order processes with time delay. For example, two blending tanks in series/parallel, two CSTRs in series with first order dynamics for each CSTR, etc., can be properly represented by second order plus time delay dynamics. Simulations were performed for critically damped and over damped second order process plus time delay dynamics described by Eq. (4). Since, many combinations for the dead time and the two process time constants were possible, the ratios τ_1/θ and τ_1/τ_2 were varied from 1 to 50. In all the simulations, τ_1 was always greater than or equal to τ_2 and the dead time was never greater than the sum of τ_1 and τ_2. Though it is always possible to approximate a SOPTD model by a FOPTD model and tuning methods based on the approximated FOPTD model can be then used, Weigand and Kegerreis [20] proved that

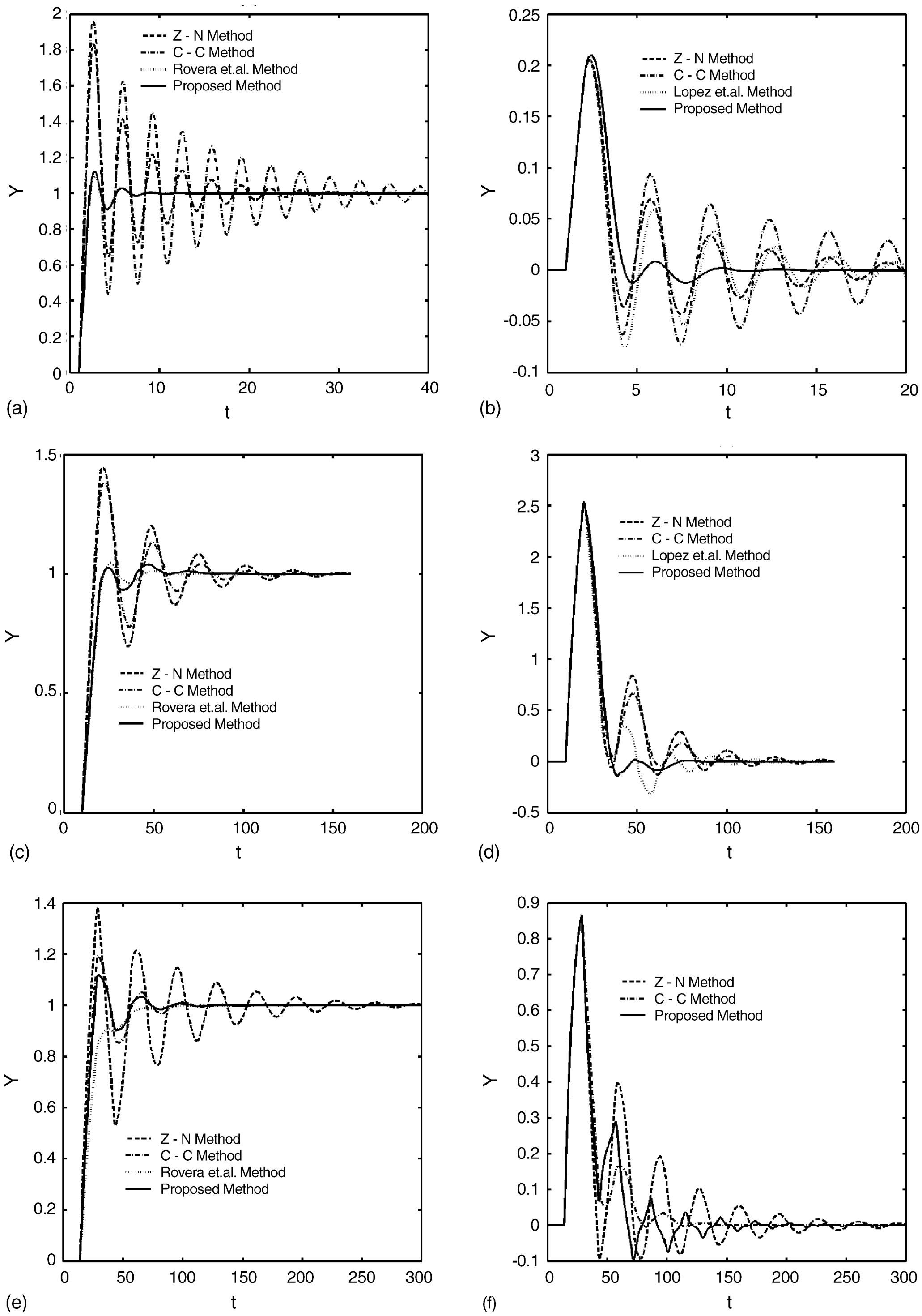

Fig. 4. Closed loop response for first order plus time delay process: (a, c and e) set point change and (b, d and f) load change for processes $G_{P1}(s)$, $G_{P2}(s)$ and $G_{P3}(s)$, respectively.

tuning methods based on full SOPTD models lead to superior results. Archibald and Tae-Won [16] performed simulations for finding the optimum tuning parameters for SOPTD processes with first order disturbance dynamics. The controller defined by Eq. (1) was used in their study. As discussed in Section 1, they have produced many charts based on different normalized process and control parameters with first order disturbance. Their results cannot be compared with the proposed method because the controller in our study has parallel configuration and also the dynamics of the disturbance variable $D(t)$, is unit step change rather than a first order exponential disturbance. Since, the optimization was performed with the initial settings obtained from Z–N method, the results obtained from the proposed method were initially compared with Z–N method only. Also, there is

Table 3
Tuning parameters and performance characteristics for FOPTD

Process	Method	Set point change							Load change			
		K_C	τ_I	τ_D	T_r	T_s	Os	IAE	K_C	τ_I	τ_D	IAE
$K_{P1}G_{P1}(s)$	Z–N	6.39	2.31	0.37	1.7	17.6	1.83	4.4	6.39	2.31	0.37	0.61
	C–C	6.92	2.27	0.35	1.7	35.7	1.97	7.7	6.92	2.27	0.35	0.88
	R/L	4.4	7.0	0.4	2.2	4.7	1.1	1.8	6.32	1.71	0.39	0.66
	P–M	4.5	7.0	0.37	2.1	4.7	1.12	1.8	5.26	1.93	0.7	0.42
$K_{P2}G_{P2}(s)$	Z–N	0.43	19.37	3.1	16.4	78.4	1.43	23.49	0.43	19.37	3.1	49.83
	C–C	0.4	18.09	3.08	16.8	66.3	1.38	21.02	0.4	18.09	3.08	46.16
	R/L	0.27	16.39	3.48	20.6	20.6	1.05	15.96	0.36	11.39	4.82	39.73
	P–M	0.28	16.7	4.33	20.0	36.8	1.04	16.28	0.34	11.4	3.56	37.59
$K_{P3}G_{P3}(s)$	Z–N	1.15	24.0	3.84	22.2	163.4	1.38	37.7	1.15	24.0	3.84	27.3
	C–C	0.88	16.78	3.88	24.5	66.8	1.19	23.7	0.88	16.78	3.88	19.3
	R/L	0.59	14.58	4.59	54.2	54.2	–	24.7	0.76	13.40	7.42	20.87
	P–M	0.8	15.6	4.28	25.6	53.7	1.11	22.3	0.82	11.91	4.19	16.05

Table 4
Proposed tuning relations for SOPTD process

Tuning parameter	Set point change	Load change
K_C	$\dfrac{0.5723}{K_P}\left(\dfrac{\theta}{\theta+\tau_1+\tau_2}\right)^{-1.0409}$	$\dfrac{0.6202}{K_P}\left(\dfrac{\theta}{\theta+\tau_1+\tau_2}\right)^{-0.9931}$
$\dfrac{\tau_I\tau_1}{\theta}$	$0.2476(\theta+\tau_1+\tau_2)\left(\dfrac{\theta}{\theta+\tau_1+\tau_2}\right)^{-1.6501}$	$(\theta+\tau_1)\left(13.81\left(\dfrac{\theta}{\theta+\tau_1+\tau_2}\right)^2-14.906\left(\dfrac{\theta}{\theta+\tau_1+\tau_2}\right)+4.566\right)$
$\dfrac{\tau_D\tau_1}{(\tau_2+\theta)\theta}$	$0.0943\left(\dfrac{\theta}{\theta+\tau_1+\tau_2}\right)^{-1.4636}$	$0.0921\left(\dfrac{\theta}{\theta+\tau_1+\tau_2}\right)^{-1.4849}$

very scarce literature on optimization of controller tuning parameters obtained for SOPTD and SOPTD with lead processes with minimization of IAE as the objective function. Furthermore, the optimal controller tuning parameters vary with the objective function used in optimization. Fig. 5 shows the validation of the tuning models with the simulation data for set point and load changes. The corresponding tuning models are shown in Table 4. The conclusions from these models were generally similar to those obtained for FOPTD models except for the derivative time. Once again the controller gain is inversely proportional to the process gain. With the increase in the dead time, the integral time decreases. Contrary to the FOPTD process, with an increase in the dead time the derivative time also increases. Fig. 6 shows the integral time to derivative time ratio as a function of fraction dead time for SOPTD processes. It clearly shows that a constant τ_I/τ_D ratio was obtained for load change while for set point change at very small fraction dead time, the τ_I/τ_D ratio increases. This result is similar to the result obtained for FOPTD process. A very similar trend was also observed by Åstrom and Hägguland [19] for lower fraction dead time.

3.4. Case studies for SOPTD

The performance of three different processes with SOPTD dynamics were analyzed by using the controller parameters obtained from the proposed method and was compared with parameters obtained from Z–N method. The process transfer functions for the three processes (with small dead time, dead time equal to the dominant process time constant and dominant dead time) are described by the following equations:

$$K_{P4}G_{P4}(s)=\frac{1}{(15s+1)(2s+1)}\exp(-2s) \qquad (14)$$

Table 5
Tuning parameters and performance characteristics for SOPTD

Process	Method	Set point change							Load change			
		K_C	τ_I	τ_D	T_r	T_s	Os	IAE	K_C	τ_I	τ_D	IAE
$K_{P4}G_{P4}(s)$	Z–N	7.44	8.25	1.32	5.5	53.0	1.77	13.32	7.44	8.25	1.32	1.73
	P–M	5.97	26.9	1.62	6.6	21.8	1.22	6.67	5.74	6.56	1.71	1.33
$K_{P5}G_{P5}(s)$	Z–N	2.04	21	3.36	16.6	87.6	1.5	24.44	2.04	21	3.36	13.04
	P–M	1.57	22.83	4.38	19.3	44.5	1.1	17.15	1.77	14.58	3.88	9.39
$K_{P6}G_{P6}(s)$	Z–N	0.88	6.75	4.32	10.5	167.4	1.65	29.65	0.88	6.75	4.32	27.21
	P–M	0.88	13.5	2.16	11.1	55.0	1.4	15.14	0.68	7.82	2.82	12.68

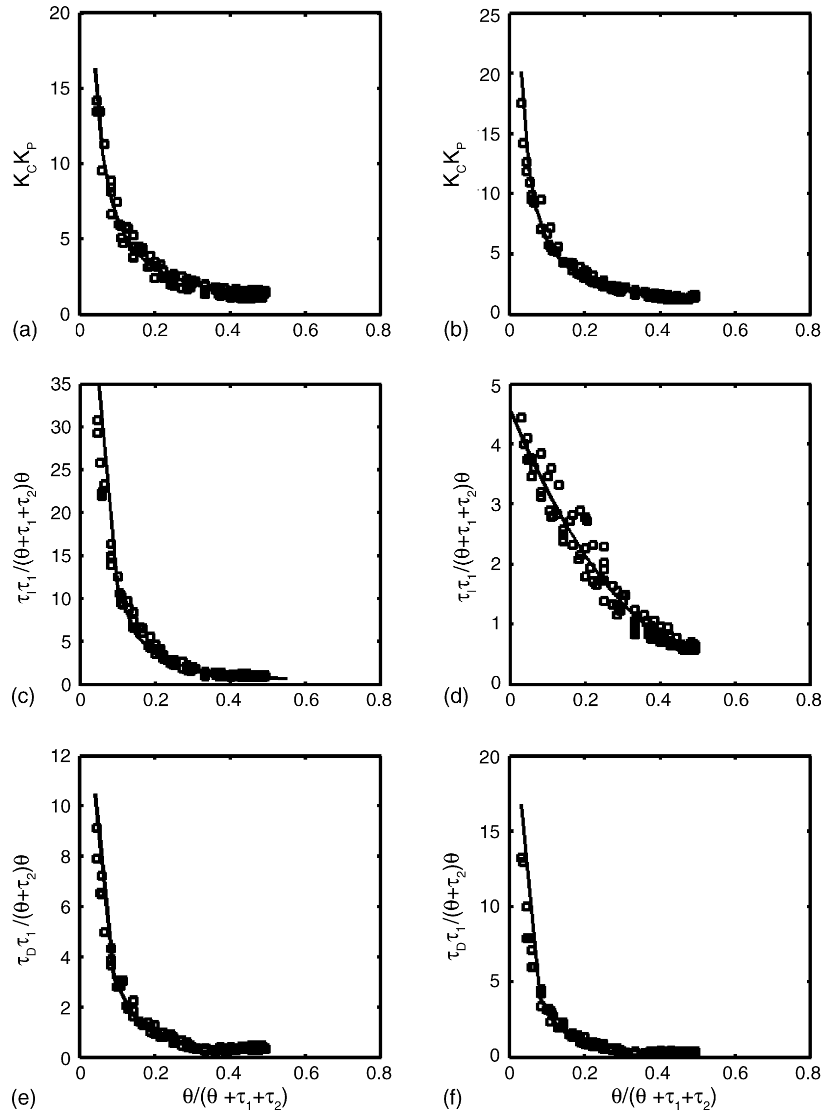

Fig. 5. Dimensionless control parameters obtained from model vs. simulation results for SOPTD (described by Eq. (4)): (a, c and e) for set point change and (b, d and f) for load change.

$$K_{P5}G_{P5}(s) = \frac{1}{(7s+1)^2}\exp(-7s) \tag{15}$$

$$K_{P6}G_{P6}(s) = \frac{2}{(5s+1)(3s+1)}\exp(-5s) \tag{16}$$

The closed loop response due to a unit step change in set point and load, respectively, are shown in Fig. 7. The controller parameters used and the corresponding performance IAE calculations are shown in Table 5. Fig. 7(e and f) shows the relative advantage of the proposed methodology over the Z–N method. The large oscillations resulting with the Z–N tuning will not only result in a high IAE but also in highly oscillatory control actions that will lead to wear of the control element. On the other hand, using the tuning parameters obtained from the proposed method would give a better performance with reduced oscillations. Table 5 shows that lower IAE values were obtained by using the proposed method of tuning. Also, the overshoot and the settling time were smaller as compared to the Z–N tuning method.

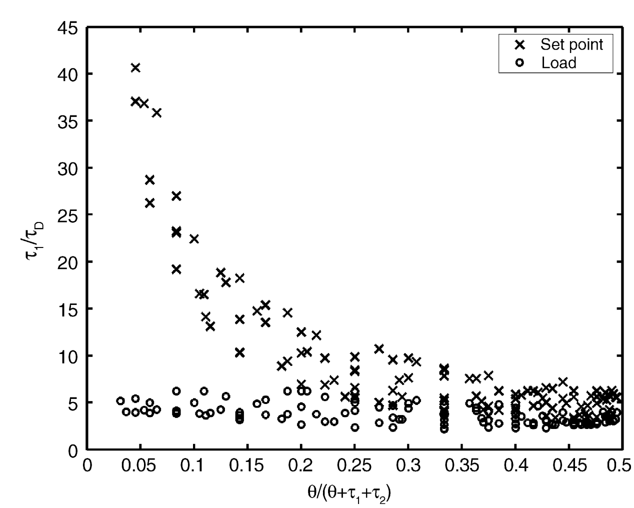

Fig. 6. The ratio between integral time and derivative time as a function of fraction dead time for SOPTD process.

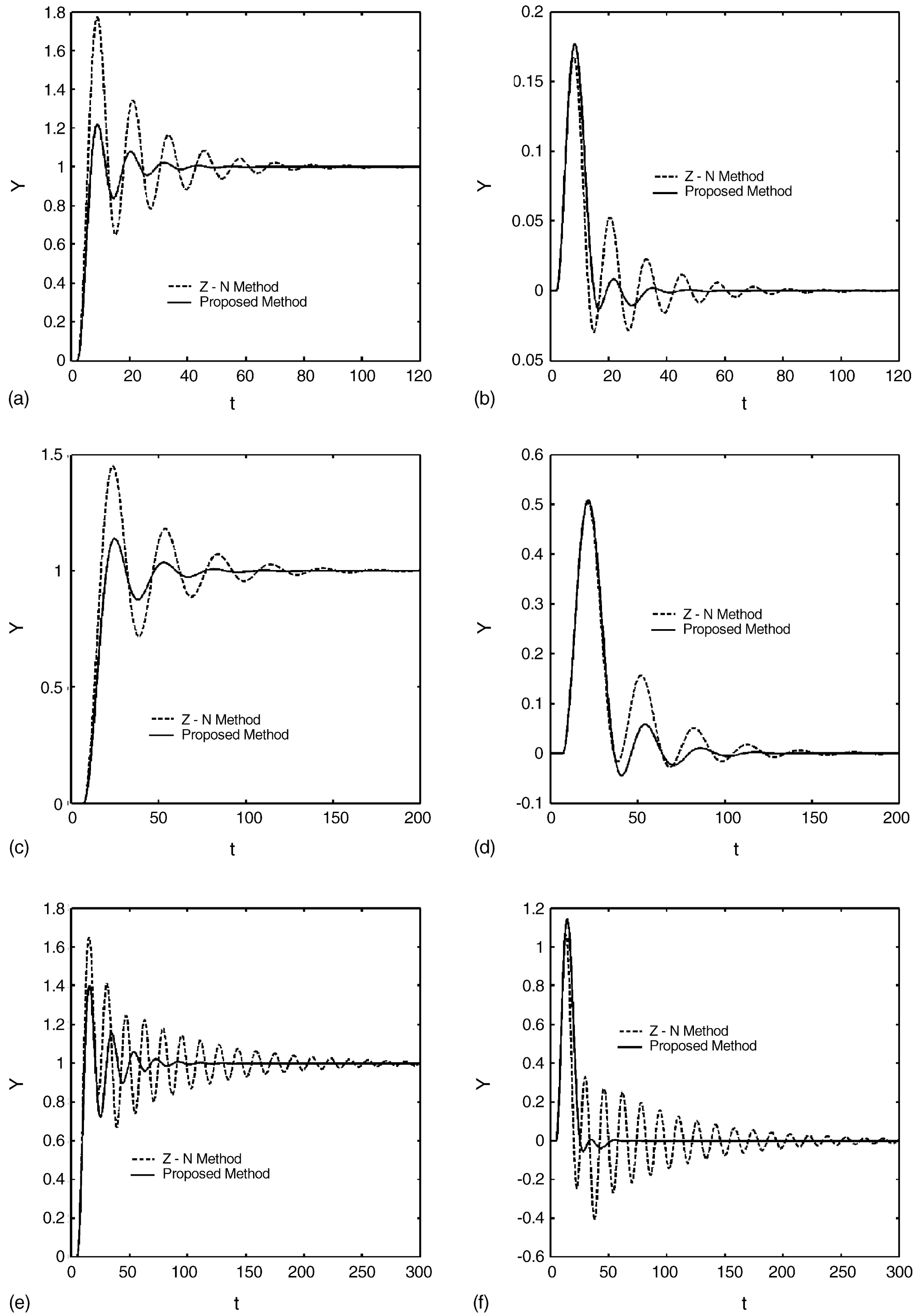

Fig. 7. Closed loop response for SOPTD: (a, c and e) set point change for processes 1, 2 and 3, respectively, and (b, d and f) load change for processes 1, 2 and 3, respectively.

3.5. SOPTDLD

Another frequently encountered process in chemical engineering is the second order plus time delay with lead. The most common examples of processes that can be represented by these types of models are processes with recycle streams (Seborg et al. [9] and Bequette [21]). To enrich product quality, recycling a part of the product stream is a conventional technique practiced by many industries. A scarce amount of literature is available on tuning PID controllers for these types of processes. Tuning the PID controller for these types of processes is a challenging task as the lead time constant (τ_3 in Eq. (5)) has very strong influence on the closed loop response. The closed loop response will vary based on the relative values of τ_1 and τ_3. In the simulations,

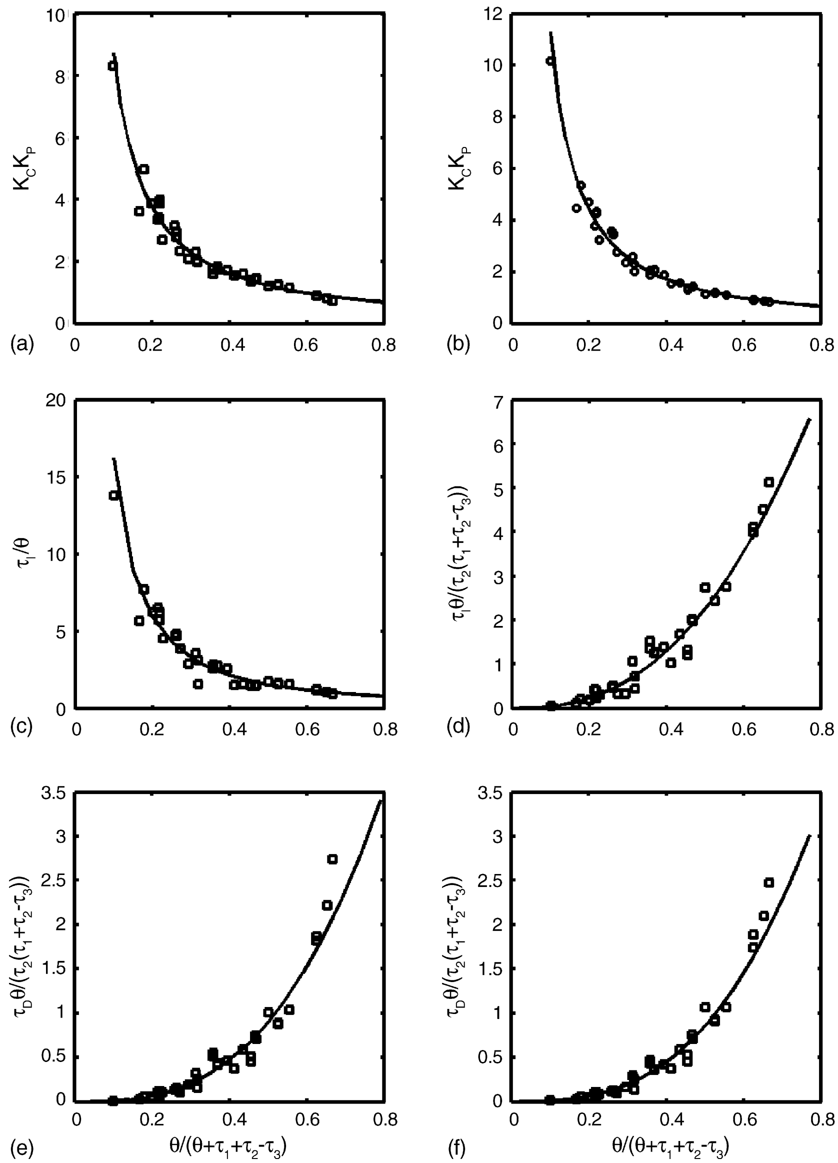

Fig. 8. Dimensionless control parameters obtained from model vs. simulation results for second order plus time delay with lead process (described by Eq. (5)): (a, c and e) for set point change and (b, d and f) for load change.

different combinations of the process parameters are considered with the range being 1–20. Furthermore, the simulations were performed such that $\tau_1 > \tau_3$. Fig. 8 shows the validity of the proposed tuning models with respect to simulation results. For SOPTDLD processes, the controller gain is inversely proportional to the process gain. The controller gain for load change is more than that of set point change. The integral time and derivative time are modeled in a similar fashion and were found to increase with a decrease in the fraction dead time. The models obtained for SOPTDLD are shown in Table 6. The τ_I/τ_D ratio as a function of fraction dead time is shown in Fig. 9. For a set point change and at very low values of fraction dead time, the

Table 6
Proposed tuning relations for SOPTDLD process

Tuning parameter	Set point change	Load change
K_C	$\dfrac{0.5254}{K_P}\left(\dfrac{\theta}{\theta+\tau_1+\tau_2-\tau_3}\right)^{-1.2206}$	$\dfrac{0.4938}{K_P}\left(\dfrac{\theta}{\theta+\tau_1+\tau_2-\tau_3}\right)^{-1.3594}$
τ_I	$0.5657\theta\left(\dfrac{\theta}{\theta+\tau_1+\tau_2-\tau_3}\right)^{-1.4569}$	$12.585\dfrac{\tau_2(\tau_1+\tau_2-\tau_3)}{\theta}\left(\dfrac{\theta}{\theta+\tau_1+\tau_2-\tau_3}\right)^{-2.4858}$
$\dfrac{\tau_D\theta}{\tau_2(\tau_1+\tau_2-\tau_3)}$	$6.7572\left(\dfrac{\theta}{\theta+\tau_1+\tau_2-\tau_3}\right)^{-2.9057}$	$6.465\left(\dfrac{\theta}{\theta+\tau_1+\tau_2-\tau_3}\right)^{-2.9185}$

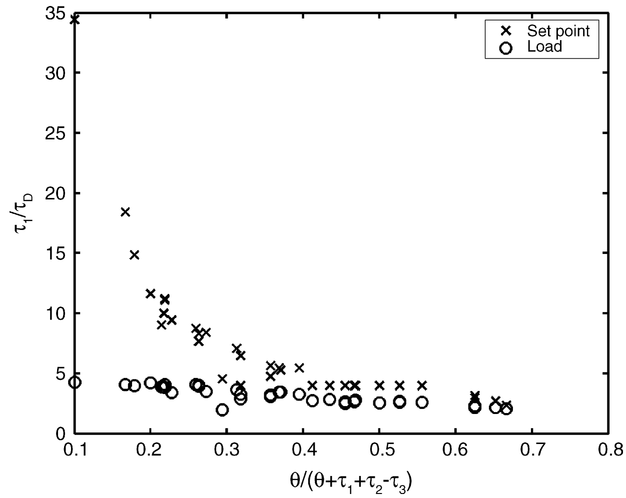

Fig. 9. The ratio between integral time and derivative time as a function of fraction dead time for SOPTDLD process.

τ_I/τ_D ratio goes to infinity. Nevertheless, for fraction dead time greater than 0.1 the τ_I/τ_D ratio is constant (around 5) for both set point and load changes, which is favorable for many industrial controllers.

3.6. Case studies for SOPTDLD

To compare the performance of the processes with the proposed model for control parameters, three processes with different process parameter configurations are analyzed. The transfer functions for the three processes $G_{P7}(s)$, $G_{P8}(s)$ and $G_{P9}(s)$ are described by Eqs. (17)–(19), respectively.

$$K_P G_{P7}(s) = \frac{6(s+1)}{(7s+1)(3s+1)} \exp(-5s) \tag{17}$$

$$K_P G_{P8}(s) = \frac{4(5s+1)}{(15s+1)(7s+1)} \exp(-10s) \tag{18}$$

$$K_P G_{P9}(s) = \frac{15(7s+1)}{(20s+1)(5s+1)} \exp(-10s) \tag{19}$$

The closed response due to step change in the set point and load are shown in Fig. 10 for the three processes, $G_{P7}(s)$, $G_{P8}(s)$ and $G_{P9}(s)$. Fig. 10 clearly shows that the response obtained by the proposed method is superior to the one obtained using Z–N settings. The controller parameters and the corresponding performance characteristics obtained for the two methods are shown in Table 7. From Table 7, we can conclude that by using the proposed method of tuning, the overshoot and the settling time are significantly reduced in addition to the minimization of IAE compared to the conventional Z–N method.

4. Robustness, performance and control effort

There are many mathematical definitions for performance and control effort in the literature. The definitions for robustness, performance and control effort used to compare the proposed method with different existing methods are similar to those used by Foley and co-workers [8]. The robustness of a control system is usually evaluated by randomly varying the process parameters while keeping the control parameters constant. This is done to study the controller performance in situations where there are uncertainties or errors in the process model. To study the robust stability for the different processes discussed in the earlier sections, the process parameters, i.e. K_P, θ, τ_1, τ_2 and τ_3 are simultaneously increased until the system becomes conditionally stable (the closed response will have sustained oscillations). Then robustness was defined as the largest deviation in the model parameters for which the response becomes conditionally stable. This is mathematically represented by the following equation:

$$\begin{aligned} \text{Robustness} = {} & \frac{K_{P\,\max} - K_P}{K_P} \times 100\% \quad \text{or} \\ & \times \frac{\tau_{1\,\max} - \tau_1}{\tau_1} \times 100\% \quad \text{or} \\ & \times \frac{\tau_{2\,\max} - \tau_2}{\tau_2} \times 100\% \quad \text{or} \\ & \times \frac{\tau_{3\,\max} - \tau_3}{\tau_3} \times 100\% \quad \text{or} \\ & \times \frac{\theta_{\max} - \theta}{\theta} \times 100\% \end{aligned} \tag{20}$$

The subscript ‘max’ refers to the value that corresponds to the conditionally stable response.

The performance is defined as follows:

$$\text{Performance} = \frac{\sigma_{e,\mathrm{M}}^2}{\sigma_e^2} \times 100\% \tag{21}$$

Table 7
Tuning parameters and performance characteristics for SOPTDLD

Process	Method	Set point change							Load change			
		K_C	τ_I	τ_D	T_r	T_s	Os	IAE	K_C	τ_I	τ_D	IAE
$K_{P7}G_{P7}(s)$	Z–N	0.39	12.6	2	9.8	42.5	1.5	13	0.39	12.6	2	32.5
	P–M	0.27	14.31	2.93	12.5	12.5	1.04	9.4	0.32	8.32	2.5	28.1
$K_{P8}G_{P8}(s)$	Z–N	0.63	22.25	3.56	17.4	73.1	1.52	23.8	0.63	22.25	3.56	35.7
	P–M	0.47	27.5	4.78	20.2	40.1	1.1	17.1	0.5477	15.04	4.1	30.53
$K_{P9}G_{P9}(s)$	Z–N	0.17	19.81	3.17	16.2	78.3	1.52	23.1	0.17	19.81	3.17	125.8
	P–M	0.12	25.8	4.53	19.8	47.1	1.0	17.2	0.14	12.41	3.79	107.5

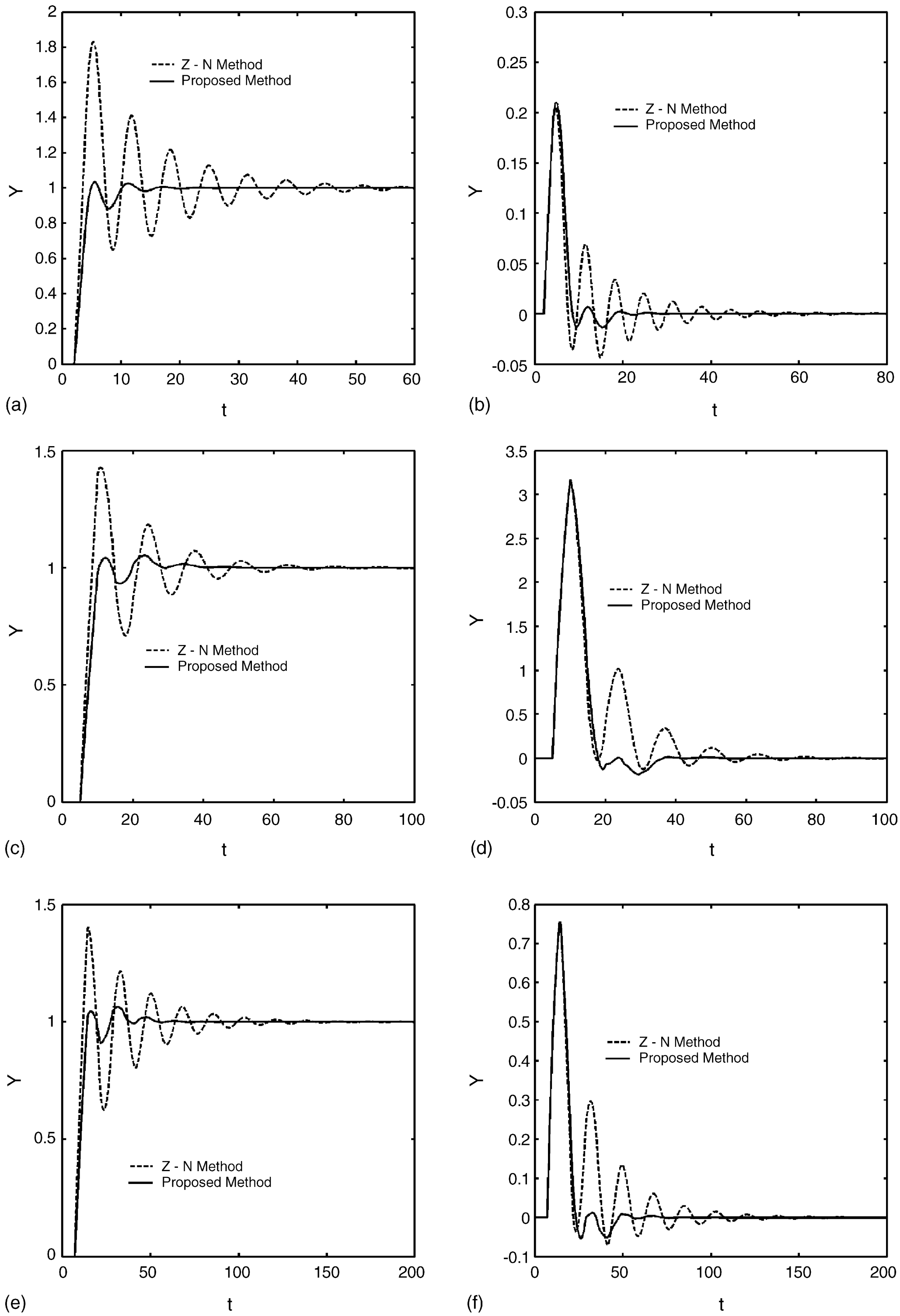

Fig. 10. Closed loop response for SOPTD with lead process: (a, c and e) set point change for processes 1, 2 and 3, respectively, and (b, d and f) load change for processes 1, 2 and 3, respectively.

where σ_e^2 is the variance of the error and $\sigma_{e,\mathrm{M}}^2$ is the minimum control error variance achieved by using parallel PID structure with $\alpha = 0.05$. The performance of PID controller, either series or parallel configuration will be bounded by 0% (unstable control system) and 100% (best control system). Similarly, the control effort is defined as the ratio of the variance of the control error, $\sigma_{\Delta U}^2$ (which also represents the valve adjustments) to the minimum variance PID controller, $\sigma_{\Delta U,\mathrm{M}}^2$. The definition for control effort is then given by the following equation:

$$\text{Control effort} = \frac{\sigma_{\Delta U}^2}{\sigma_{\Delta U,\mathrm{M}}^2} \times 100\% \tag{22}$$

Table 8
Comparison of robustness, performance and control effort for different processes corresponding to a unit step change in set point

Process	Method	Set point change		
		Robustness	Performance	Control effort
$K_{P1}G_{P1}(s)$	Z–N	22	82.9	143.5
	C–C	11	63.26	196.92
	P–M	94	99.43	106.3
$K_{P2}G_{P2}(s)$	Z–N	35	99.21	103.03
	C–C	43	99.6	102.1
	P–M	88	99.47	101.67
$K_{P3}G_{P3}(s)$	Z–N	31	97.29	111.58
	C–C	57	99.75	104.79
	P–M	65	99.86	104.79
$K_{P4}G_{P4}(s)$	Z–N	27	83.97	145.49
	P–M	83	97.18	119.11
$K_{P6}G_{P5}(s)$	Z–N	36	97.19	112.4
	P–M	75	99.52	109.37
$K_{P6}G_{P6}(s)$	Z–N	33	86.53	152.53
	P–M	38	98.91	105.45
$K_{P7}G_{P7}(s)$	Z–N	41	99.6	101
	P–M	97	100	101
$K_{P8}G_{P8}(s)$	Z–N	40	99.2	103.5
	P–M	87	99.99	102
$K_{P9}G_{P9}(s)$	Z–N	37.5	99.7	99.96
	P–M	94	100.9	100.6

Table 8 shows the comparison of the robustness, performance and control effort calculated for different processes using different control parameters obtained from various tuning methods for unit step changes in the set point. The results show that the control system designed by using the proposed method is highly robust compared to those obtained by using other methods. In almost all case studies (except for $K_{P6}G_{P6}(s)$), the robustness measure obtained with the proposed method is over 85%. The robustness achieved with other methods is very low and this would entail frequent tuning of the PID parameters for model uncertainties and changes in levels of operating conditions. The control effort using the proposed method was always lower than those obtained for other methods, but relatively the difference was not high except for some cases. Small control effort would cause smooth increments in the control valve.

5. Conclusions

PID tuning relations (with parallel form) for FOPTD, SOPTD and SOPTD with lead processes are developed for unit step change in set point and load (separately). The derivative filter factor α was preset to 0.1 in all simulations. The tuning relations were derived based on the minimization of IAE. Generalized tuning rules for PID controller, as functions of the process parameters were developed. The proposed tuning rules were compared with conventional tuning methods. For FOPTD process, the tuning rules were developed for θ/τ_1 ratio varying from 0.1 to 2 while other conventional methods can be used only up to a ratio of 1. Tuning rules for critically damped and over damped SOPTD processes were developed. Different case studies for different FOPTD, SOPTD and SOPTD with lead process were analyzed. From the comparison of the results obtained using the proposed tuning method and various other methods for tuning PID controllers, it can be concluded that the proposed tuning method gives a better response in all cases. Finally, the robust stability, performance and control effort were evaluated for the case studies with unit step changes in the set point. It was shown that the controller tuned with the proposed method is highly robust and ensures best achievable controller performance, compared to other conventional tuning methods. Also, the proposed sets of PID tuning rules are simple, accurate and efficient for practitioners.

Acknowledgements

The financial support of the Natural Sciences and Engineering Research Council (NSERC) of Canada is gratefully acknowledged.

Appendix A. Nomenclature

D	load (or disturbance) variable
e	control error, R–Y
G_C	feedback controller transfer function
$G_{C,Ser}$	servo compensator
G_P	process transfer function
K_C	proportion gain of PID controller
K_P	gain of the process
Os	overshoot
r	set point
s	Laplace operator
t_{ss}	time to reach steady state
T_r	rise time
T_s	settling time (to reach 5% of the steady state value)
U	controller output
Y	controlled variable (or process variable)

Greek symbols

α	derivative filter factor
θ	dead time
σ_e^2	control error variance
$\sigma_{e,M}^2$	minimum error variance achievable using parallel PID control with $\alpha = 0.05$
$\sigma_{\Delta U}^2$	variance of controller output (also manipulative variable)
$\sigma_{\Delta U,M}^2$	variance of controller output using parallel PID control with $\alpha = 0.05$
τ_D	derivative time of the PID controller
τ_I	integral time of the PID controller
τ_1	process time constant
τ_2	process time constant
τ_3	process time constant

Subscripts

max max value for which the response becomes conditionally stable

1–9 indices to represent different processes

References

[1] J.G. Ziegler, N.B. Nichols, Optimum settings for automatic controllers, Trans. ASME 64 (1942) 759–768.

[2] G.H. Cohen, G.A. Coon, Theoretical consideration of related control, Trans. ASME 75 (1953) 827–834.

[3] D. Chen, D.E. Seborg, PI/PID controller design based on direct synthesis and disturbance rejection, Ind. Eng. Chem. Res. 41 (19) (2002) 4807–4822.

[4] D.E. Rivera, M. Morari, S. Skogestad, Internal model control. 4. PID controller design, Ind. Eng. Chem. Process Des. Dev. 25 (1986) 252–265.

[5] A. Andrášik, A. Mészáros, S.F. de Azevedo, On-line tuning of a neural PID controller based on plant hybrid modelling, Comput. Chem. Eng. 28 (2004) 1499–1509.

[6] C.H. Lee, C.C. Teng, Calculation of PID controller parameters by using a fuzzy neural network, ISA Trans. 42 (3) (2003) 391–400.

[7] Y. Yeong-Koo, T.I. Kwon, A neural PID controller for the pH neutralization process, Ind. Eng. Chem. Res. 38 (3) (1999) 978–987.

[8] W. Micheal, M.W. Foley, R.H. Julien, B.R. Copeland, A comparison of PID controller tuning methods, Can. J. Chem. Eng. 83 (August) (2005) 712–722.

[9] D.E. Seborg, T.F. Edgar, D.A. Mellichamp, Process Dynamics and Control, second ed., Wiley, New York, 2004.

[10] M.L. Luyben, W.L. Luyben, Essentials of Process Control, McGraw-Hill, New York, 1997.

[11] F.G. Shinskey, Process Control Systems, third ed., McGraw-Hill, New York, 1988.

[12] W.L. Luyben, Process Modeling, Simulation and Control for Chemical Engineers, McGraw-Hill, New York, 1973.

[13] R. Ciancone, T. Marlin, Tune controllers to meet plant objectives, Control 5 (1992) 50–57.

[14] A.B. Corripio, Tuning of Industrial Control Systems, Instrument Society of America, 1990.

[15] T. Sayeed, T. Mahdi, Optimal tuning of PID controllers for first order plus time delay models using dimensional analysis, in: The Fourth International Conference on Control and Automation (ICCA'03), June 2003, pp. 942–946.

[16] G.H. Archibald, K. Tae-Won, Effect of disturbance dynamics on optimum control of second order plus dead time processes, Instrum. Chem. Petrol. Ind. 20 (April) (1988) 69–79.

[17] A.A. Rovira, Ph.D. Dissertation, Louisiana State University, Baton Rouge, 1981.

[18] A.M. Lopez, P.W. Murrill, C.L. Smith, Controller tuning relationships based on integral performance criteria, Instrum. Technol. 14 (November) (1967) 57.

[19] K.J. Ǻstrom, T. Hägglund, Revisiting the Ziegler–Nichols step response method for PID control, J. Process Control 14 (2004) 635–650.

[20] W.A. Weigand, J.E. Kegerreis, Comparison of controller-setting techniques as applied to second-order dead time processes, Ind. Eng. Chem. Process Des. Dev. 11 (1) (1972) 86–90.

[21] B.W. Bequette, Process Control—Modeling, Design and Simulation, Prentice Hall, 2003.

[22] T. Marlin, Process Control, Mc Graw-Hill, NY, 1995.